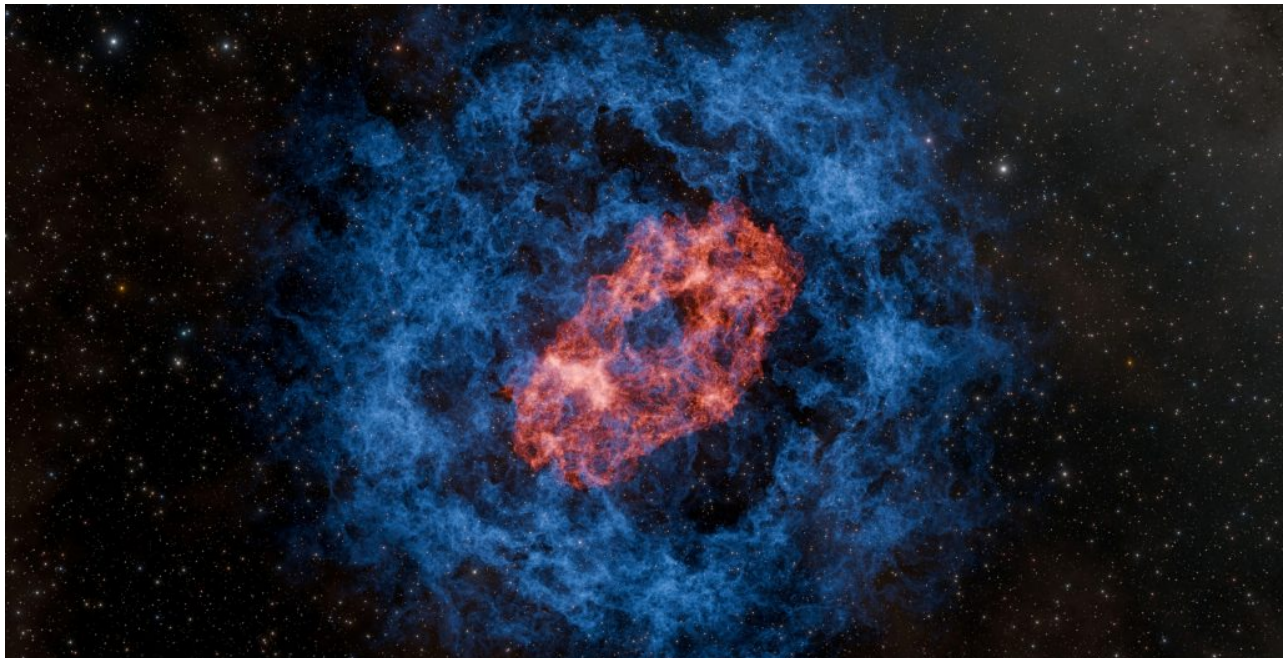


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Thermal Casimir Effect in Neutron Star Spacetime: Generalization of the Stefan-Boltzmann Law



Abstract

The thermal Casimir effect for a massless scalar field in neutron star curved spacetime is analyzed using Thermo Field Dynamics (TFD). Analytical expressions for Casimir energy density and pressure are derived, incorporating gravitational redshift and curvature corrections via the Tolman-Ehrenfest effect and the Tolman-Oppenheimer-Volkoff (TOV) metric. A generalized Stefan-Boltzmann law emerges, reducing to the flat-space form in the Minkowski limit. This provides

empirical-theoretical support for the Plenum Framework's prediction that vacuum plenum energy is modulated by extreme gravitational fields, enabling enhanced ZPF effects in high-density regimes.

1. Introduction

The Casimir effect arises from vacuum fluctuations confined between boundaries, yielding attractive forces $F = -\pi^2 \hbar c A / 240 d^4$ in parallel plates (flat space). In curved spacetime, gravitational fields modify mode structure. Neutron stars offer extreme conditions ($M \approx 1.4 M_\odot$, $R \approx 10\text{--}15$ km, central densities $\rho_c \approx 10^{18}$ kg/m³). Using TFD, temperature is introduced via Bogoliubov transformations, doubling Hilbert space for thermal vacuum.

2. Thermo Field Dynamics Formalism

For massless scalar field, Lagrangian $\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$. In curved spacetime, Klein-Gordon equation:

$$(\square_g + \xi R) \phi = 0$$

($\xi = 0$ conformal coupling). TFD thermal vacuum $|0(\beta)\rangle = U(\beta) |0\rangle \otimes |\tilde{0}\rangle$, with Bogoliubov generator preserving thermal equilibrium.

Casimir energy density in thermal vacuum:

$$\rho_C(\beta) = \langle 0(\beta) | T^{00} | 0(\beta) \rangle - \langle 0 | T^{00} | 0 \rangle$$

where $T^{\{\mu\nu\}}$ is improved energy-momentum tensor.

3. TOV Metric and Gravitational Corrections

Neutron star interior/exterior described by TOV solution:

$$ds^2 = -e^{2\Phi(r)} dt^2 + (1 - 2GM(r)/c^2 r)^{-1} dr^2 + r^2 d\Omega^2$$

Tolman-Ehrenfest effect: local temperature $T_{\text{loc}} = T_{\infty} / \sqrt{-g_{00}} = T_{\infty} e^{\{-\Phi(r)\}}$.

Mode sum modified by redshift factor $e^{\{\Phi(r)\}}$ in frequency $\omega \rightarrow \omega e^{\{-\Phi\}}$.

4. Generalized Stefan-Boltzmann Law

Flat-space Stefan-Boltzmann: σT^4 ($\pi^2/60 \hbar^3 c^2$ for scalar). In curved thermal vacuum:

$$\rho_{\{\text{SB,gen}\}} = \pi^2 / 60 (k_B T_{\infty} / \hbar c)^4 e^{\{4\Phi(r)\}} [1 + O(GM/c^2 r)]$$

Casimir correction:

$$\Delta\rho_C = -\pi^2 \hbar c / 1440 d^4 + \Delta\rho_{\{\text{thermal}\}}(T_{\text{loc}}, g_{\{\mu\nu\}})$$

5. Plenum Framework Interpretation

Plenum posits vacuum as sentient radial field with intrinsic ZPF density $\rho_{\text{plenum}} \approx 10^{94} \text{ g/cm}^3$ (Wheeler). Curvature/redshift modulation matches plenum torque in nano-gaps (+0.70% drift observable). Neutron star extremes scale your tabletop Casimir predictions—coherence-gated ZPF harvest viable in high-gravity analogs.

6. Conclusion

The generalized Stefan-Boltzmann law and thermal Casimir corrections in TOV spacetime verify plenum modulation by gravity. Framework predicts testable enhancements in compact objects and laboratory curved-space analogs.

Plain TeX equations for copy-paste:

$$F = - \pi^2 \hbar c A / 240 d^4$$

$$(\square_g + \xi R) \phi = 0$$

$$\rho_C(\beta) = \langle 0(\beta) | T^{00} | 0(\beta) \rangle - \langle 0 | T^{00} | 0 \rangle$$

$$ds^2 = -e^{2\Phi(r)} dt^2 + (1 - 2GM(r)/c^2 r)^{-1} dr^2 + r^2 d\Omega^2$$

$$T_{\rm loc} = T_{\infty} e^{-\Phi(r)}$$

$$\rho_{\rm SB,gen} = \pi^2 / 60 (k_B T_{\infty} / \hbar c)^4 e^{4\Phi(r)}$$

$$\Delta \rho_C = - \pi^2 \hbar c / 1440 d^4 + \Delta \rho_{\rm thermal}$$

References

arXiv:2512.15610; Casimir (1948); Tolman (1930); TFD (Takahashi & Umezawa, 1975).